Indian Statistical Institute, Bangalore

B. Math.(Hons.) II Year, First Semester

Semestral Examination

Analysis -III (Back Paper)

Time: 3 hours ———- Instructor: B.Rajeev

1. Show that the volume of the ellipsoid $E = \{(x, y, z) : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\}$ $a, b, c > 0$ is $\frac{4}{3}\pi$ abc. [12]

2. Calculate $\int\int_S f(x, y)dydx$ where $S = \{(x, y) : x \geq 0, y \geq 0,\ 0 \leq x+y \leq 2\}$ and $f(x, y) = e^{\frac{x-y}{x+y}}$ [12]

3. Let $T = [0\ \pi] \times [0, \frac{\pi}{2}]$ and $\overrightarrow{r}(u, v) = a\ \cos u\ \cos v \overrightarrow{i} + a\ \sin u\ \cos v \overrightarrow{j} + a\ \sin v \overrightarrow{k}$.

   a) Is $\overrightarrow{r}(u, v)$ a 1-1 function from T to the upper hemisphere? Prove your answer. [4]

   b) Show that the north pole is the only singular point, i.e., the only point at which the fundamental vector product vanishes. [7]

4. Show (without using spherical coordinates) that area of the upper hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$ is $2\pi\ a^2$. [10]

5. Let $\overrightarrow{r}(u, v), (u, v) \in T$ and $\overrightarrow{R}(s, t), (s, t) \in T'$ be smoothly equivalent representations of a surface S over the regions T and T' respectively. If $f : S \longrightarrow \mathbb{R}$ is a continuous and bounded function then show that

   $\int\int_{R(T)} fdS = \int\int_{R(T^1)} fdS.$ [15]